1st Junior Penchick Online Mathematical Olympiad

Qualifying Stage, Paper E

(Ages 18 and below)

9 May 2026

INSTRUCTIONS AND INFORMATION

1. **There are 20 questions to be answered in 90 minutes, split into two parts.** The highest possible score for this contest is 48 points.
   - *Part I (multiple-choice):* Each question has four choices (A, B, C, and D), and only one of these choices is correct. Submit the best answer to each question. Two points are given for a correct answer, for a total of 24 points for this section.
   - *Part II (short answer):* Submit the best answer to each question. Three points are given for a correct answer, for a total of 24 points for this section. For this paper, all answers are **integers** from 0 to 999 inclusive.
2. **Always exhibit academic honesty during the contest.** Do not use calculators, protractors, graph paper, search engines, artificial intelligence, and other external applications, resources, or help while answering the contest. Rulers, compasses, and blank or lined paper are allowed. Cheating in any form is strictly prohibited and will be acted upon based on the violation system.
3. Note that diagrams are **not necessarily drawn to scale**.
4. The top 60% of contestants in this paper will be invited to take the **1st JPOMO Final Stage** on 27 June 2026.

Enjoy the contest!

Part I: Multiple-Choice Questions

Each question has four choices (A, B, C, and D), and only one of these choices is correct. Submit the best answer to each question. Two points are given for a correct answer, for a total of 24 points for this section.

1. We are given the sequence $(1,\ 2026,\ 3,\ 2022,\ 5,\ 2018,\ 7,\ 2014,\ \ldots)$, where the odd-numbered terms follow the odd positive integers and the even-numbered terms follow $2026-4n$ (where n is a non-negative integer). What is the remainder when the sum of the first 2763 terms is divided by 1000?

(a) 270 (b) 560 (c) 50 (d) 865

2. Let the folowing equations hold for real numbers x, y:

$$\begin{aligned} &\log_2(x^4y) = \log_4(x^2y^6), \\ &\log_8(xy) = 1, \\ &x+y = 2^a + 2^b \end{aligned}$$

If a and b are rational numbers such that $a > b$, then find $50a+5b$.

(a) 96 (b) 67 (c) 105 (d) 69

3. Consider a grid of points with integer coordinates (x,y). A path starts at the bottom-left corner $(0,0)$ and moves to the top-right corner $(6,3)$ using only steps to the right $(1,0)$ or upward $(0,1)$. A path is called *penchicky* if it never crosses the lines

$$y = x \quad \text{and} \quad y = 2x-1,$$

though it may touch squares on those lines. Determine the number of penchicky paths from $(0,0)$ to $(6,3)$.

(a) 14 (b) 15 (c) 48 (d) 84

4. Let $\triangle ABC$ have side lengths $a=40$, $b=37$, and $c=13$. Find the length of the altitude to side a.

(a) 11 (b) 12 (c) 13 (d) 14

5. A group of 6 penchicks are taking a class photo. There are 15 seats in a row, but since the penchicks are socially awkward, no two can sit on adjacent seats. How many possible seatings are there, considering all penchicks are indistinguishable?

(a) 180 (b) 70 (c) 210 (d) 105

6. Let N be a four-digit number in base b, written as 1434_b. When N is converted to base 10, it is divisible by $b+5$. Find the sum of all integer bases b for which this is true.

(a) 24 (b) 48 (c) 51 (d) 56

7. A spring oscillates according to the function

$$y = 3\sin\left(\frac{\pi t}{4}\right) + 2\cos\left(\frac{\pi t}{2}\right),$$

where y is the displacement in centimeters and t is time in seconds. Determine the maximum displacement of the spring in centimeters.

(a) $\frac{21}{8}$ (b) $\frac{127}{64}$ (c) $\frac{83}{32}$ (d) $\frac{41}{16}$

8. A penchick is modelled as a solid formed by joining two right circular conical frustums, stacked vertically. The upper frustum has top radius 3 cm, bottom radius 5 cm, and vertical height 6 cm. The lower frustum has top radius 5 cm, bottom radius 7 cm, and vertical height 6 cm. The circular face where the two frustums meet is internal. If the total volume of the penchick model can be written in the form $x\pi$, find the value of x.

Note: A frustum is the solid formed when the top is cut off a cone by a plane parallel to its base.

(a) 316 (b) 523 (c) 446 (d) 218

9. When five fair six-sided dice are rolled, the probability that the sum of the outcomes is 17 can be expressed as $\frac{p}{q}$ for coprime positive integers p, q. Find $p+q$.

(a) 1473 (b) 2054 (c) 713 (d) 780

10. Renren chose a number $k > 1$ and wrote the sequence (a_i) on a whiteboard, defined by $a_1 = 1$ and $a_{n+1} = 2a_n + n$ for $n \geq 1$. The evil Penrick came and replaced every number with its remainder divided by k. He then found out there exists some t such that $a_t = a_{t+1} + 1$. Given that $k \leq 100$, how many different possible numbers k could Renren have chosen?

(a) 9 (b) 10 (c) 11 (d) 12

11. In $\triangle ABC$ with $BC = 28$, points P and Q lie on sides AB and AC, respectively, such that

$$AP = 75, AQ = 72, PQ = 21, \text{and } PC = 35.$$

Given that the quadrilateral $PBCQ$ has an incircle that is tangent to all 4 sides and has area $x\pi$, find x.

(a) 105 (b) 98 (c) 162 (d) 144

12. Let ω be a complex number such that

$$\omega^7 = 1 \quad \text{and} \quad \omega \neq 1.$$

Define

$$S = \prod_{k=1}^{6} \left(1 + \omega^k + \omega^{2k}\right).$$

What is the exact value of S?

(a) $-i$ (b) -1 (c) i (d) 1

Part II: Short-Answer Questions

Submit the best answer to each question. Three points are given for a correct answer, for a total of 24 points for this section.
For this paper, all answers are **integers** from 0 to 999 inclusive.

1. In the official Penchick language, sentences are formed by concatenating the words `Squawk`, `Chirp`, and `PenPen`. However, Blizzy's computer is broken, so Blizzy can only type one of the following blocks at any step:

`Squawk Chirp`, `Chirp PenPen`, `PenPen`.

Blizzy cannot delete words. If a valid sentence consists of exactly 8 words, how many valid sentences exist?

2. Let a and b be positive integers such that

$$\gcd(a,b) = 12.$$

Suppose that

$$\gcd(3a+2b,\ 5a+7b) = 84.$$

What is the smallest possible value of $a+b$?

3. Let $\triangle ABC$ have $AB = 14$, $AC = 48$ and $BC = 50$. Let the circumcircle of $\triangle ABC$ be ω. Let A' be a point on ω such that $\overline{AA'}$ is perpendicular to $\overline{BC}$. Also, let A'' be on ω such that AA'' is a diameter of ω. If the length of $A'A''$ can be written in the form $\frac{m}{n}$, where m, n are relatively prime positive integers, then what is $m - 10n$?

4. Let the sequence (a_n) be defined recursively by

$$a_1 = 1, \quad a_{n+1} = a_n + n^2 \quad \text{for } n \geq 1.$$

Compute the sum

$$\sum_{n=1}^{15} a_n.$$

5. Given 753300031 has 12 factors, find the last 3 digits of its largest prime factor.

6. Two circles Γ_1 and Γ_2 intersect at points A and B. Their centers are O_1 and O_2, with radii $O_1A = 13$ and $O_2A = 15$. The distance $O_1O_2 = 14$. Let the common external tangents to the circles meet at T. Then, $TA = \frac{a\sqrt{b}}{c}$, where a, b, c are positive integers, b is square-free, and $\gcd(a, c) = 1$. Find $100a + 10b + c$.

7. A dart is thrown uniformly at random at a circular target of radius 100. The target is divided into 100 rings by circles of radii $1, 2, 3, \ldots, 100$, each one fully in all circles which are larger. Starting from the center, the rings are alternately assigned a score of $+1$ and -1, with the innermost disk assigned $+1$. The expected value of the score of a random throw can be expressed as $\frac{m}{n}$, where m and n are relatively prime integers and n is positive. Find $m + n$.

8. Two circles of radius 7 are such that the centre of each circle lies on the other circle. A square is inscribed in the space between the circles. If the area of the square can be written in the simplest form $\frac{a(b - \sqrt{c})}{d}$ where a, b, c, d are positive integers, find $a + b + c + d$.

Answer Key

Part I: Multiple-Choice Questions

1. a
2. a
3. c
4. b
5. c
6. c
7. d
8. a
9. c
10. c
11. d
12. d

Part II: Short-Answer Questions

1. 171
2. 36
3. 804
4. VOID (4215)
5. 261
6. VOID (1851)
7. 99
8. 62